Durham University
Mathematical Sciences Department
MSc Particles, Strings and Cosmology
Standard Model

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Homework

1 Higgs Decay

1. Compute the Feynman rules for the coupling of the physical Higgs boson to pairs of W^+W^- and Z^0Z^0 bosons.

2. Compute the Feynman rules for the coupling of the physical Higgs boson to fermion-antifermion pairs.

3. Calculate the decay rate for the Higgs boson into W^+W^-, Z^0Z^0 and fermion-antifermion pairs.

Solution

1. The gauge kinetic term of Higgs is given by

$$\mathcal{L} \supset \mathcal{L}_{\text{Higgs-kin.}} = (D_\mu H)^\dagger (D^\mu H), \tag{1}$$

where

$$D_\mu = \partial_\mu - i\frac{g'}{2}B_\mu - i\frac{g}{2}W^a_\mu \sigma^a, \tag{2}$$

in the fundamental representation, and

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}. \tag{3}$$

The covariant derivative of the field H is

$$D_\mu H = \frac{1}{\sqrt{2}} \begin{pmatrix} -i\frac{g}{2}(W^1_\mu - iW^2_\mu)(v+h) \\ \partial_\mu h + \frac{i}{2}(gW^3_\mu - g'B_\mu)(v+h) \end{pmatrix}. \tag{4}$$

Hence,

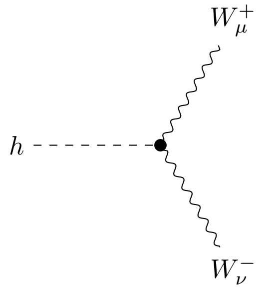
$$\mathcal{L}_{\text{Higgs-kin.}} = \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \frac{1}{8}g^2(v+h)^2 W^+_\mu W^{-\mu} + \frac{1}{8}\sqrt{g^2+g'^2}(v+h)^2 Z^0_\mu Z^{0\mu}, \tag{5}$$

where

$$W^\pm_\mu = \frac{W^1_\mu \mp iW^2_\mu}{\sqrt{2}} \qquad \text{and} \qquad Z^0_\mu = \frac{gW^3_\mu - g'B_\mu}{\sqrt{g^2+g'^2}}. \tag{6}$$

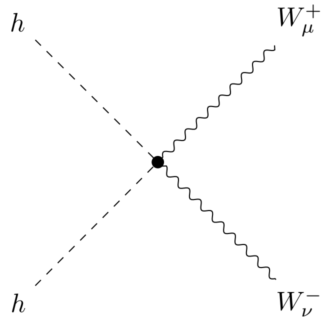
We have 4 types of interactions between the weak bosons and Higgs boson,

$hW^+W^-:$



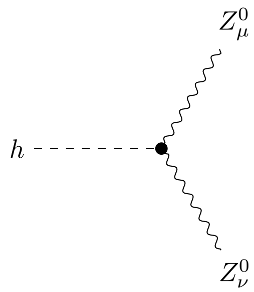
$$= 2i\frac{m_W^2}{v}g_{\mu\nu} \tag{7}$$

$hhW^+W^-:$



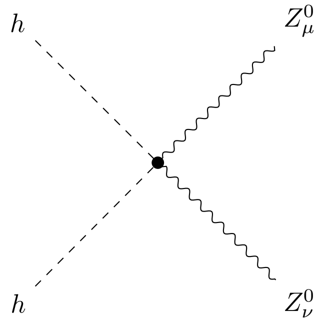
$$= 2i\frac{m_W^2}{v^2}g_{\mu\nu} \tag{8}$$

$hZ^0Z^0:$



$$= 2i\frac{m_Z^2}{v}g_{\mu\nu} \tag{9}$$

$hhZ^0Z^0:$



$$= 2i\frac{m_Z^2}{v^2}g_{\mu\nu} \tag{10}$$

where

$$m_W = \frac{vg}{2} \qquad \text{and} \qquad m_Z = \frac{v\sqrt{g^2+g'^2}}{2}. \tag{11}$$

This can be easily seen from expanding $\mathcal{L}_{\text{Higgs-kin.}}$.

2. The field H is $SU(2)_L$ doublet. For gauge-invariance, it must couple with one $SU(2)_L$ doublet fermion and one $SU(2)_L$ singlet fermion. Hence,

$$\mathcal{L}_{Yukawa} \supset -y_f\bar{f}_R H F_L - y_f \bar{F}_L H f_R, \tag{12}$$

where y_f is a dimensionless constant. This is a generalised form to choose the fermions (leptons and quarks). Simplify the Lagrangian by multiplying the fields, then

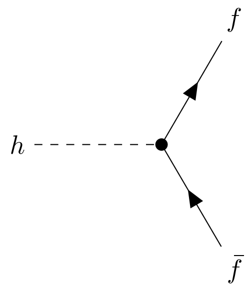
$$\mathcal{L}_{Yukawa} \supset -\frac{y_f v}{\sqrt{2}}\bar{f}f - \frac{y_f}{\sqrt{2}}h\bar{f}f, \tag{13}$$

with a mass

$$m_f = \frac{y_f v}{\sqrt{2}}. \tag{14}$$

Hence, the interaction term between Higgs boson and fermion-antifermion pairs is

$h\bar{f}f$:



$$= -i\frac{m_f}{v} \tag{15}$$

3. The differential decaying rate formula is given by from Peskin & Schroeder textbook is given by

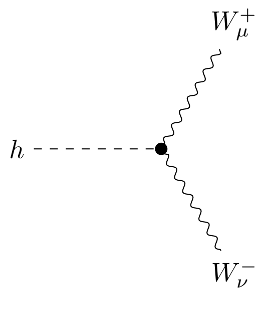
$$d\Gamma = \frac{1}{2m_{\mathcal{A}}}\left(\prod_f \frac{d^3 p_f}{(2\pi)^3}\right)|\mathcal{M}(m_{\mathcal{A}} \to \{p_f\})|^2 (2\pi)^4 \delta^{(4)}(p_{\mathcal{A}} - \Sigma p_f). \tag{16}$$

The problem asks to calculate the decay rate for the Higgs boson into W^+W^-, Z^0Z^0 and fermion-antifermion pairs. We have three decays:

$\boldsymbol{h \to W^+W^-}$

The first order matrix element is

$i\mathcal{M}_{hWW} =$



$$= \epsilon_\mu^*(p, r_1)\left(2i\frac{m_W^2}{v}g^{\mu\nu}\right)\epsilon_\nu(k, r_2), \tag{17}$$

so

$$\sum_{\text{spin}} |\mathcal{M}|^2_{hWW} = \left(\frac{2m_W^2}{v}\right)^2 \left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{m_W^2}\right)\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_W^2}\right), \tag{18}$$

and hence,

$$\sum_{\text{spin}} |\mathcal{M}|^2_{hWW} = \left(\frac{2m_W^2}{v}\right)^2 \left(2 + \frac{(p\cdot k)^2}{m_W^4}\right). \tag{19}$$

From the face that

$$m_h^2 = (p+k)^2 \Rightarrow p\cdot k = \frac{1}{2}(m_h^2 - 2m_W^2), \tag{20}$$

then

$$\sum_{\text{spin}} |\mathcal{M}|^2_{hWW} = \left(\frac{m_h^2}{v}\right)^2 \left(1 - 4\frac{m_W^2}{m_h^2} + 12\frac{m_W^4}{m_h^4}\right). \tag{21}$$

Now, the decay rate is given by[1]

$$\Gamma_{hWW} = \frac{(2\pi)^4}{2m_h}\left(\sum_{\text{spin}} |\mathcal{M}|^2_{hWW}\right)\int\left(\frac{d^3p}{(2\pi)^3(2E_p)}\frac{d^3k}{(2\pi)^3(2E_k)}\right)\delta^{(4)}\left(m_h - p - k\right). \tag{22}$$

Since we are in the rest frame of h, then the 4-dimensional delta function can be written as

$$\delta^{(4)}(m_h - p - k) = \delta(m_h - E_p - E_k)\delta^{(3)}(-\mathbf{p} - \mathbf{k}), \tag{23}$$

Hence, integrate over the 3-dimensional delta function,

$$\Gamma_{hWW} = \frac{1}{2m_h(2\pi)^2}\left(\sum_{\text{spin}} |\mathcal{M}|^2_{hWW}\right)\int \delta(m_h - 2E_p)\left(\frac{d^3p}{4E_p^2}\right). \tag{24}$$

Convert to the spherical coordinates and then change the integration variable to the energy, then

$$\Gamma_{hWW} = \frac{1}{2m_h(4\pi)}\left(\sum_{\text{spin}} |\mathcal{M}|^2_{hWW}\right)\int \delta(m_h - 2E_p)\left(\frac{|\mathbf{p}|dE_p}{E_p}\right), \tag{25}$$

and evaluate over the delta function,

$$\Gamma_{hWW} = \frac{|\mathbf{p}|}{8\pi m_h^2}\left(\sum_{\text{spin}} |\mathcal{M}|^2_{hWW}\right). \tag{26}$$

One can calculate the momentum as follows:

$$\frac{1}{2}m_h = E_p = \sqrt{|\mathbf{p}|^2 + m_W^2} \Rightarrow |\mathbf{p}| = \frac{m_h}{2}\sqrt{1 - \left(\frac{2m_W}{m_h}\right)^2}. \tag{27}$$

Finally, substituting this result gives

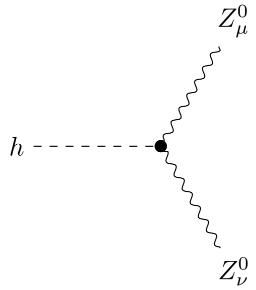
$$\boxed{\Gamma_{hWW} = \frac{1}{16\pi}\frac{m_h^3}{v^2}\sqrt{1 - \frac{4m_W^2}{m_h^2}}\left(1 - \frac{4m_W^2}{m_h^2} + 12\frac{m_W^4}{m_h^4}\right).} \tag{28}$$

[1] Use the rest frame of the field h.

$h \to Z^0 Z^0$

The first order matrix element is

$$i\mathcal{M}_{hZZ} =$$



$$= \epsilon^*_\mu(p, r_1) \left(2i\frac{m_Z^2}{v} g^{\mu\nu}\right) \epsilon_\nu(k, r_2), \tag{29}$$

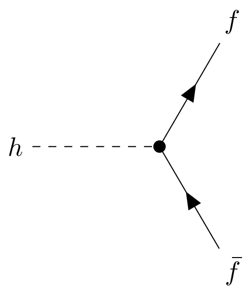
Now, this case has the same steps as the decaying from Higgs boson to the W-weak-boson. However, since the products are identical, then we have to multiply the answer by a factor 1/2. Hence,

$$\boxed{\Gamma_{hZZ} = \frac{1}{32\pi}\frac{m_h^3}{v^2}\sqrt{1 - \frac{4m_Z^2}{m_h^2}}\left(1 - \frac{4m_Z^2}{m_h^2} + 12\frac{m_Z^4}{m_h^4}\right).} \tag{30}$$

$h \to \bar{f} f$

The first order matrix element is

$$i\mathcal{M}_{h\bar{f}f} =$$



$$= \bar{u}_f(k_1, s_1)\left(-i\frac{m_f}{v}\right) v_{\bar{f}}(k_2, s_2). \tag{31}$$

so

$$|\mathcal{M}|^2_{h\bar{f}f} = \left(\frac{m_f}{v}\right)^2 (\bar{u}_1 v_2)(\bar{v}_2 u_1). \tag{32}$$

Sum over s_1 and s_2, and the colors (if existed),[2]

$$\sum_{\text{color}}\sum_{\text{spin}} |\mathcal{M}|^2_{h\bar{f}f} = n_c \left(\frac{m_f}{v}\right)^2 \mathrm{Tr}[(\not{k}_2 - m_f)(\not{k}_1 - m_f)] = 4n_c \left(\frac{m_f}{v}\right)^2 (k_1 \cdot k_2 - m_f^2), \tag{33}$$

where n_c is the color factor, i.e.

$$n_c = \begin{cases} 1, & \text{for liptons,} \\ 3, & \text{for quarks.} \end{cases} \tag{34}$$

Using the rest frame of the field h,

$$m_h = (k_1 + k_2)^2 = k_1^2 + k_2^2 + 2k_1 \cdot k_2 = 2m_f^2 + 2k_1 \cdot k_2, \tag{35}$$

[2] The trace of an odd number of gamma matrices vanishes.

which implies

$$k_1 \cdot k_2 - m_f^2 = \frac{m_h^2}{2} - 2m^2. \tag{36}$$

Thus

$$\sum_{\text{color}} \sum_{\text{spin}} |\mathcal{M}|^2_{h\bar{f}f} = 2n_c \left(\frac{m_f m_h}{v}\right)^2 \left(1 - \left(\frac{2m_f}{m_h}\right)^2\right). \tag{37}$$

The two-particle differential decay for two final states with the same masses is given by

$$d\Gamma = \frac{|\mathcal{M}|^2}{64\pi^2 m_{\mathcal{A}}} \sqrt{1 - \frac{4m_f^2}{m_{\mathcal{A}}^2}} d\phi d(\cos\theta). \tag{38}$$

Hence,

$$d\Gamma_{h\bar{f}f} = n_c \frac{m_h}{32\pi^2} \left(\frac{m_f}{v}\right)^2 \left(1 - \left(\frac{2m_f}{m_h}\right)^2\right)^{3/2} d\phi d(\cos\theta). \tag{39}$$

Integrate both sides,

$$\boxed{\Gamma_{h\bar{f}f} = n_c \frac{m_h}{8\pi} \left(\frac{m_f}{v}\right)^2 \left(1 - \left(\frac{2m_f}{m_h}\right)^2\right)^{3/2}.} \tag{40}$$

Finished